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Forest Value and Optimal Rotations in Continuous Cover Forestry

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Abstract The Faustmann forest rotation model is a celebrated contribution in economics. The model provides a forest value expression and allows a solution to the optimal rotation problem valid for perpetual rotations of even-aged forest stands. However, continuous forest cover forest management systems imply uneven-aged dynamics, and while a number of numerical studies have analysed specific continuous cover forest ecosystems in search of optimal management regimes, no one has tried to capture key dynamics of continuous cover forestry in simple mathematical models. In this paper we develop a simple, but rigorous mathematical model of the continuous cover forest, which strictly focuses on the area use dynamics that such an uneven-aged forest must have in equilibrium. This implies explicitly accounting for area reallocation and for weighting the productivity of each age class by the area occupied. We present results for unrestricted as well as area-restricted versions of the models. We find that land values are unambiguously higher in the continuous cover forest models compared with the even-aged models. Under area restrictions, the optimal rotation age in a continuous cover forest model is unambiguously lower than the corresponding area restricted Faustmann solution, while the result for the area unrestricted model is ambiguous.

Keywords Capital budgeting · Faustmann rotation model · Uneven-aged forest management

JEL Classification Q23

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1 Introduction

Over the recent decades, storms, forest health decline, forest fire and pest issues as well as climate change have raised concerns about the sustainability of much of the plantation forestry in Europe—in particular with regard to coniferous species. Therefore, forest managers and decision-makers have looked for an alternative management paradigm to provide more stable forests and better support for the ecosystem services that forests provide, including biodiversity protection, clean water, erosion control and carbon sequestration. In many countries and regions it has been advocated that this alternative is found in what has been termed near-natural or continuous cover forestry (Spiecker et al. 2004), and around the turn of the millennium this change in the paradigm was officially adapted in, e.g. Wales (National Assembly for Wales 1999), Niedersachsen (FNiedersächsisches Forstplannungsamt 1995) and Denmark (Ministry of the Environment 2002).

For these reasons, the economics of the conversion from coniferous to deciduous tree species and from even-aged to continuous cover[1] forests has been subject to several empirical and numerical case analyses (e.g. Buongiorno 2001; Hanewinkel 2001; Knoke and Plusczyk 2001; Price 2003; Tarp et al. 2000; Wollborn 2000; Price and Price 2006; Tahvonen 2009). These numerical analyses have tried to capture the effects of issues like smaller regeneration costs, potentially larger harvesting and management costs, different assortments (harvest diameters), and the more complex forest dynamics of continuous cover forestry (CCF). Case-specific effects on value production as well as decision parameters like thinning rules and rotation ages have also been discussed and analysed numerically.

The basis for such analyses is the celebrated Faustmann formula attributed to the German forester Faustmann (1849), who discussed the correct valuation of a piece of land in forestry use. A few years later Pressler (1995) published similar ideas. This soil expectation value has long been widely accepted as the only correct criterion for maximisation of the value of an even-aged forest with a perpetuity of identical rotations of forest. It was solved correctly for the first time by the Swedish economist Ohlin (1995), though Samuelson (1976) suggests that Faustmann's writings indicate he knew of the correct solution too. This paper is the first to present and analyse a model of how we can extend the Faustmann–Pressler–Ohlin framework to encompass and shed light on the effects of one of the significant differences between continuous cover forestry and the even aged Faustmann forestry: the different ways in which land is utilized by the trees of different age classes in the forest.

One assumption of the Faustmann formulation sometimes questioned is that of a perpetuity of identical rotations. Alternative assumptions are, however, easily handled as shown by e.g. Navarro (2003) who show how to adjust for alternative uses of forested land in the future. Another issue often addressed in the literature due to the long time horizon and infrequent cash flows at forest stand level is the possible requirements for particular levels of cash flows (typically more or less even flows). Under certain circumstances a normal forest (with equal areas in each even-aged age class) is optimal and supports an even cash flow (Heaps 1984; Salo and Tahvonen 2002). In some countries the area allocated to forestry and other land uses is in practice fairly fixed or even given by law and not easily changed. This violates the assumption of free land and capital markets and raises the question whether maximising the value under the assumption of an area restricted situation deviates substantially from the unrestricted case of CCF as we know it does in the Faustmann case. If we assume no substitutability between land and capital, maximisation of the economic return under an area

[1] We use the term "continuous cover". "Uneven-aged" is also used frequently in the literature—mostly as a direct synonymous.

restriction provides a solution, where the optimal rotation age is independent of the interest rate, and equal to the maximum sustainable yield from a given area[2] (Reed 1986).

A key feature of the Faustmann model and the associated normal forest concept is that once a mature stand, and hence a tree, is harvested, the land area and resources (access to soil minerals, nutrients, light, precipitation etc.) released are, from an economic point of view, allocated entirely to regenerating a stand of trees of the youngest age class in the forest. In the CCF models, the un-even and multi-storey stands, and in particular the resulting spatial and vertical structures of the stands, imply different dynamics of area use. This has been pointed out by silviculturists as a potentially strong cause of increase in economic productivity.[3] It is this intriguing aspect of CCF that we focus on here.

When analysing the economics of CCF, different forms of numerical diameter-class or single-tree growth models have been used. Several authors extend the Faustmann approach for optimal rotation age to CCF by assuming that the aim is to maximise the present value of returns from a present state to infinity (e.g. Buongiorno 2001; Jacobsen et al. 2013; Knoke and Plusczyk 2001; Schou et al. 2012). Adams and Ek (1974) were among the first to describe continuous cover forest stands by a diameter-dependent matrix model and modelling of growth, and thereby making the calculation and optimization of the value of s CCF numerically manageable. Adams and Ek (1974) look at the optimal stand structure, not the optimal rotation age, though this is indirectly included. Haight (1985) extends this problem to include optimal rotation age, and since then similar numerical approaches have been extensively used for various variants of CCF (e.g. Buongiorno 2001; Haight 1985, 1990; Heshmatol Vaezin et al. 2009; Jacobsen and Helles 2006; Price and Price 2006; Pukkala and Kolström 1988; Tahvonen 2009; Meilby and Nord-Larsen 2012; Rämö and Tahvonen 2015).

So far, there have been only few attempts to analyse rigorously and analytically the various aspects of continuous cover forestry relative to even-aged forests (Kuuluvainen et al. 2012). However, no successful attempts to include the difference in how area is utilized at stand level into algebraically solvable models of the forest valuation principle have been made. Samuelson (1976) mentioned the importance of including area utilization, but did not analyse this further. In an extensive review Newman (2002) did not mention models with the area utilization across age classes. Nautiyal (1983) did include area in his algebraic description of the maximisation problem in CCF, but assumed that area utilization is independent of tree age and therefore it does not influence the results.

Within fisheries economics, there have also been attempts to handle uneven-aged structures, through year-class models and cohort models, starting with the paper of Beverton and Holt (1957). However there are important differences to the characteristics of the resource: 1) In fishery models, harvest is the control variable, whereas in forestry it is the rotation age, 2) growth in fishery depends on the stock, whereas in forestry it depends on time, and 3) fish moves around and can thereby utilize unoccupied space. This causes that the spatial dynamic differs. Furthermore, there are differences in terms of management practice and consequently the focus of how to optimize: in fishery, regeneration takes place by (naturally) spawning, whereas in forestry it may also take place through planting (see Conrad and Clark 1991); and selectivity of gears when harvesting is central in year-class models in fishery,

[2] This only holds under the conditions of no regeneration costs and no price-dependency on tree size. Two simplifying assumptions we maintain in this paper.

[3] Morsing (2001) and as early as Muus (1921) mention the importance of distribution of area utilization between trees and the fact that in uneven-aged management more space can be occupied by larger trees that may have a higher value growth per area and time unit, relative to the Faustmann normal forest case. However, little empirical evidence exist (O'Hara and Nagel 2006) which can attribute the difference solely due to different structures.

whereas selectivity can be made perfectly in forestry (see Neher 1990). Finally, the focus differs in forestry, where no market failure due to a stock externality is present. Therefore, in forestry focus is on optimal extraction from a private owner's perspective, whereas in fishery the focus is on welfare economic optimum and regulation hereof. Consequently, while there are similarities to the literature of fishery, the question we address here of area utilization is not addressed in the fishery economic literature.

In this paper we aim to describe analytically the economic implications of incorporating area utilization in a forest rotation framework. We do so by suggesting a parallel formulation to the Faustmann formula fitting the CCF case, where the dynamics of area utilization in the forest stand are taken into account. We also derive expressions for the optimal rotation age. Methodologically the approach in this paper is similar to that of Chang (1981), but with important extensions of model assumptions, which allow us to produce new results that explicitly consider area utilization across age classes occupying the same forest stand. We look at the optimal rotation age and area use within each forest stand of a forest, both with and without restrictions on the total area of the forest and compare the expressions with the Faustmann and maximum sustainable yield solutions. Including area utilization in a way where it is endogenous to management has not been handled in the forest valuation and management literature before.

The paper is organized as follows. In the next section we elaborate on the problem in question and provide some simple arguments illustrating the importance of the different dynamics of area utilization that we include in this paper. Furthermore, we describe here some of the basic assumptions. Section 3 contains different models for the continuous cover and Faustmann cases and in Sect. 4 we provide a numerical illustration of the impact that this new approach may have on the economics of forest management. We conclude with a discussion of results and potential roads ahead.

2 The Problem in Focus

Figure 1 illustrates a basic difference between an even-aged forest as in the Faustmann rotation model and the CCF that we investigate in this paper.

As illustrated in Fig. 1 an implicit assumption in the classic Faustmann rotation model is that the single final-harvest tree is allocated a constant area throughout its life. Hence, the age-dependent production function for a single tree is related to a constant area—irrespective of tree age. As a consequence the value of the well-known normal forest in the Faustmann model is simply the sum of the present values of the different age classes, each occupying a constant area in perpetuity. Figure 1 also show that in the classic Faustmann case, when we harvest an old tree the entire area it occupies is utilized by a new, small tree.[4]

We explore the consequences of replacing this assumption with an alternative assumption, which captures the dynamics of CCF, but nevertheless remains a strong simplification, as is the Faustmann model. The basic assumption is that in the CCF, each age class only utilise the area needed for an adequate development over its lifetime.[5] Ass shown in Fig. 1 this has two consequences: (1) the area allocated to an age class depends on its age, so that the productivity is weighted by the area use of the age class (2) when an age class is harvested

[4] Notice that the classic Faustmann model and also our interpretation here are neglecting thinnings—of evenaged trees. In reality these may play a substantial role for the area utilization, and the setup can be expanded to that cover that situation, but at a loss of mathematical tractability in particular in the CCF case.

[5] We do not discuss the issues of competition, crown development and stem growth in more detail here. By 'adequate' we mean to imply a development known to result in healthy growth and good stem quality.

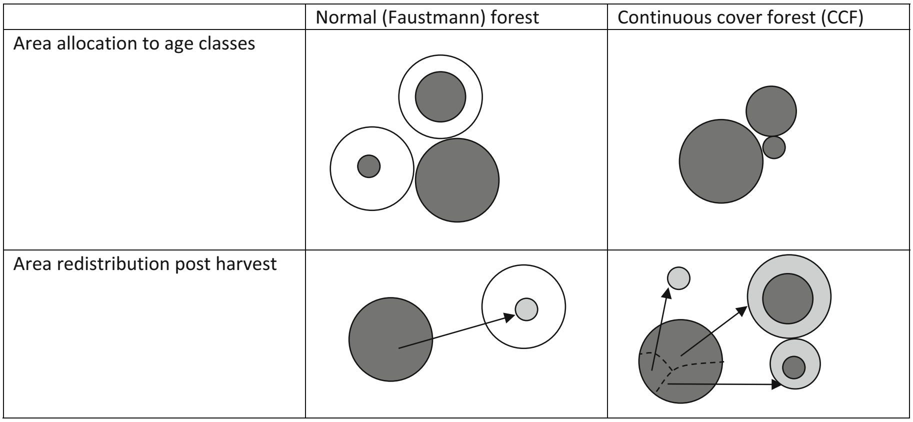

Fig. 1 Stylised illustration of the area distribution across age classes and the re-allocation of area post-harvest to remaining trees shown for both the forest systems. *Dark grey* shows current area utilised by existing trees. *Light grey* shows area about to be utilised through growth into the area opened after cutting a large tree. *Empty areas in circles* shows land allocated to, but not utilised by an age class. In the normal (Faustmann) forest there is a large area not being used by the younger trees, whereas in the CCF the age class only utilize the area needed. Thus there is a more efficient area re-allocation

the area is re-allocated to all younger age classes using the released area. Thereby the CCF obtains a more efficient area use expressed through area-reallocation after harvest compared to the Faustmann model.

We also assume that age is the basic control variable in the model[6] and we ignore thinning and regeneration costs. This allows us to focus on the pure effect of area allocation on optimal rotation age and the value of land in forest utilization. We furthermore simplify the problem by making the following assumptions:

(a) Once a tree is harvested, trees of younger age encroach immediately into the released area and occupy the released area completely.
(b) The volume growth of the individual tree over its lifetime is identical in the continuous cover and the even-aged forest.
(c) We ignore mortality, though this can be interpreted into the model quite easily as accounted for in the area taken up by an age class.
(d) We assume that only the oldest trees are harvested.[7]

Notice that the difference in area utilization between the CCF and the Faustmann model is the only difference we focus on, keeping all else constant. It is of course a simplification to assume that space is immediately used by other trees as trees cannot pull up their roots and move around. However, idle land will be taken over eventually and we thus simplify away a possible delay in this.

[6] As did Faustmann and many others. Alternatives could be size, diameter or value, but as we like Faustmann assume an identical age-size-value relationship for all individual trees—and across the forest types, age is as appropriate as size.

[7] By this assumption we avoid the need for multiple state variable models, and this eases the comparison of the models. We discuss the consequence of this assumption later.

3 Algebraic Models and Results

In this section we present four models, starting with the classic Faustmann model, deriving the well-known results (Sect. 3.1). After that, in Sect. 3.2, we impose an area restriction on the Faustmann model, and solve for the optimal rotation age for a forest containing all age classes, e.g. as in the normal forest (Heaps 1984), which we use as a benchmark. The optimality conditions for this formulation, turns out to be the maximum sustained yield solution as also mentioned in Neher (1990) and Möhring (2001). After that we develop a similar model for CCF, i.e. an area restricted CCF model and derive the optimal rotation age (Sect. 3.3). Last, we relax the area restriction for the continuous cover model and we also derive the optimal rotation age for this case in Sect. 3.4. In Sect. 3.3 we also compare a Faustmann model and a CCF model with an area restriction while the two models without an area restriction are compared in Sect. 3.4.

3.1 The Traditional Faustmann Model

Without a restriction on how much land to allocate to forestry, we can derive the optimal rotation age for each stand of trees and find the optimal area of the forest and the optimal forest value. Thus, in the traditional Faustmann rule, ignoring establishment cost and thinnings, we maximize the present value of the return from one rotation taking into account that once trees is harvested, the land is used for another identical rotation. Assuming indefinite time horizon we get the following problem[8]:

$$Max\, V_F = Max[e^{-rT}(m(T)a(T) + V_F)] = Max\left[\frac{e^{-rT}}{1-e^{-rT}} m(T)a(T)\right] \tag{1}$$

where T is the rotation age, r is the discount rate, $m(T)$ is the value per area unit of a tree that is harvested at time T, $a(T)$ is the area utilized by trees of age T and released by harvesting at time T, V_F is the soil expectation value.

In (1) $m(T)a(T)$ represent the value of trees harvested at T. Because the area allocated to each age class is constant, it equals the usual formulation with just a single function (see e.g. Neher 1990). However, in Sects. 3.2 and 3.3 we develop models with an area restriction, and to allow for comparison it is useful to distinguish the area of an age class, $a(T)$, and the value per area unit of an age class, $m(T)$ already now. We assume that $m'(T) > 0$ and $m''(T) < 0$ within the relevant range of the age of trees and that area utilization of an age class is constant over the life time of the trees so that $a(t) = a(T)$ and thereby $a'(t) = 0$ for all t.

In the middle expression in (1) the first term represent the present value of trees harvested at T ($e^{-rT}m(T)a(T)$) while the last term is the present value of all future rotations ($e^{-rT}V_F$). The first two expressions in (1) can be solved for V_F, and the solution to this problem is the last expression in (1). This expression shows that the value of a forest in a Faustmann model can be represented as an annuity with a value on $m(T)a(T)$ paid with fixed time intervals and $\frac{e^{-rT}}{1-e^{-rT}}$ is normally labelled the perpetuity factor. Finally, following Samuelson (1976) V_F can be considered as the value of bare land for forestry production.

[8] To ease interpretation, we use a slightly different notation from that in Faustmann's original paper.

The first-order condition for maximization of the last expression in (1) is:

$$V_F' = \frac{(1 - e^{-rT})(-re^{-rT} m(T)a(T) + m'(T)a(T)e^{-rT} + a'(T)m(T)e^{-rT}}{(1 - e^{-rT})^2} - \frac{re^{-rT} e^{-rT} m(T)a(T)}{(1 - e^{-rT})^2} = 0 \tag{2}$$

Rearranging (2) and using that e^{-rT} cancels out gives:

$$m'(T)a(T) + a'(T)m(T) = rm(T)a(t) + \frac{rm(T)a(T)e^{-rT}}{(1 - e^{-rT})} \tag{3}$$

Using that $a'(T) = 0$ implies that (3) can be written as:

$$\frac{m'(T)}{m(T)} = r\left[1 + \frac{e^{-rT}}{1 - e^{-rT}}\right] \tag{4}$$

Equation (4) states that at T the relative marginal value growth of the stand must equal the capital cost of the stand. This capital cost of the stand can be interpreted as the opportunity cost of the capital tied up in the current stand of trees (r) and the opportunity cost of capital for the future stands $\left(r\frac{e^{-rT}}{1-e^{-rT}}\right)$. Below we refer to the right hand side of (4) as the Faustmann capital cost of forest production assuming infinite time.

3.2 The Faustmann Model with an Area Restriction and a Normal Forest as Starting Point

The Faustmann rule gives the value of bare land for production of a single stand. As a continuous cover forest in equilibrium consists of an entire forest with all age classes represented, it is useful, for comparison, to investigate an entire normal forest consisting of T age classes[9] on a given area, A. Thus, the area is restricted to A covered by a normal forest. We can formulate an area restricted Faustmann model as:

$$Max\, V_{FAR} = Max\left[\int_O^T \frac{e^{-r(T-t)} m(T)a(T)}{1 - e^{-T}} dt\right] \tag{5}$$

$$\text{s.t } Ta(T) \leq A \tag{6}$$

where A is the total area available for the forest. Note that $e^{-r(T-t)}$ is used in the nominator in (5) when discounting, instead of e^{-rT} as in (1), because of the assumption of an initial normal forest with all trees present. Thereby a tree of t years is harvested $T - t$ years from now.

We solve the integral in (5) and the objective function reduces to:

$$Max\, V_{FAR} = Max\left[\frac{m(T)a(T)}{r}\right] \tag{7}$$

As the area restriction is binding (6) may be written as:

$$a(T) = \frac{A}{T} \tag{8}$$

[9] In a normal forest an equal area is allocated to all T age classes and therefore T becomes both the number of age classes and the rotation period.

Inserting (8) into (7) yields the following problem:

$$Max V_{FAR} = Max \left[\frac{m(T)A}{rT} \right] \tag{9}$$

The first-order condition is:

$$V'_{FAR} = \frac{m'(T)\, ArT - m(T)\, Ar}{(rT)^2} = 0 \tag{10}$$

(10) may be written as:

$$\frac{m'(T)}{m(T)} = \frac{1}{T} \tag{11}$$

Thus, at the optimal rotation age the proportional increase in value production on an area unit equals a proportional decrease in the area available for each age class. The result in (11) is a variant of the maximum sustainable yield solution (see Neher 1990), with the difference being that M here captures value production and not only biophysical production. Thus, at T the average value $\left(\frac{m(T)}{T}\right)$ is equal to the marginal value $[m\prime(T)]$. Notice that the interest rate has dropped out in (11) and this implies that no substitution possibilities between land and capital exist.

In order to compare (11) with CCF with an area restriction it is useful to write the optimality rule in an alternative way. With a binding restriction a Lagrange-function may be set up based on (6) and (7):

$$L = \frac{m(T)a(T)}{r} - \lambda_{FAR}\,(Ta(T) - A) \tag{12}$$

where λ_{FAR} is the shadow price of one unit of area and this can be interpreted as the marginal opportunity cost of land in the Faustmann model. The first-order condition with respect to T can be derived and this condition may be written as:

$$\frac{m'(T)}{m(T)} = \frac{r\lambda_{FAR}}{m(T)} \tag{13}$$

By comparing (11) and (13) we see that, at the optimal T, the marginal opportunity cost of land,λ_{FAR}, is equal to the average present value of production per unit of land:

$$\lambda_{FAR} = \frac{m(T)}{rT} \tag{14}$$

3.3 CCF with an Area Restriction

Like in the Faustmann normal forest, the continuous cover forest also have all age classes represented, but importantly age classes do not take up the same amount of land, and they are mixed in between each other all over the forest area. However, the starting point is still that harvesting is "normal", i.e. the same amount harvested each year. Furthermore, in a CCF model, land use dynamics imply that once we harvest a mature age class and start a new rotation (at age $t = T$) the area and thus growth space liberated is allocated entirely to all age classes according to the growth in area, $a'(t)$, of each age class and including the space taken up by the newest generation, $a(0)$. We assume that $a'(t) \geq 0$. Note that at any t the change in area utilization of trees in all age classes is exactly equal to the area released, $a(T)$.

The cash flow resulting from this intermediate production from the land, $a(T)$, occurs as the age classes that take this area mature and are harvested. Assuming that the entire forest covers an area A and again assuming the same production function for all age-classes, $m(t)$, we can write the maximization problem of the area-restricted CCF as:

$$Max\, V_{CAR} = Max\left[\frac{m(T)a(T)}{r}\right] \tag{15}$$

$$\text{s.t.} \int_0^T a\,(t)\; dt \le A \tag{16}$$

Equations (15) and (16) are similar to (5) and (6) except for the fact that the area utilized by an age class, $a(t)$, is now a function of age and not constant as in Sect. 3.2. We note that a nice feature of this model is that when $a(t) = a(T)$, (15) and (16) are identical to (5) and (6). With respect to the value of a forest in the models with an area restriction [(5)–(6) and (15)–(16)], we note that $\int_0^T a\,(t)\,dt \le Ta(T)$. Therefore, the value of a forest area is larger in the continuous cover model than in the Faustmann model ceteris paribus.

With a binding area restriction, (16), the Lagrange-function becomes:

$$L = \frac{m(T)a(T)}{r} - \lambda_{CAR}\left(\int_0^T a(t)dt - A\right) \tag{17}$$

where λ_{CAR} is the marginal opportunity cost of land in the CCF model.

The first-order condition is given as:

$$\frac{\partial L}{\partial T} = \frac{m'(T)a(T) + a'(T)m(T)}{r} - \lambda_{CAR}a(T) = 0 \tag{18}$$

Now (18) may be written as:

$$\frac{m'(T)a(T) + a'(T)m(T)}{a(T)m(T)} = \frac{\lambda_{CAR}r}{m(T)} \tag{19}$$

The interpretation of (19) is identical to the interpretation of (13) and for a constant $a(T)$ and thereby $a'(T) = 0$ the two equations are identical. Thus, as in Sect. 3.2, the total value growth is equal to the opportunity cost of land times the interest rate divided by the value per area unit at T. Using (17) and the fact that the two models are identical under Faustmann assumptions on $a(T)$, we can deduce that λ_{CAR} is the marginal opportunity cost of land as in Sect. 3.2, Thus, the shadow price is given by:

$$\lambda_{CAR} = \frac{m(T)a(T)}{r\int_0^T a\,(t)\,dt} \tag{20}$$

By comparing (20) with (14) we see that $\lambda_{FAR} < \lambda_{CAR}$ as we know from (16) that the value of an average unit of area is higher in the CCF model than in the Faustmann model $\left(\frac{1}{T} < \frac{a(T)}{\int_0^T a(t)dt}\right)$. Therefore we obtain that $\frac{m'(T)a(T)+a'(T)m(T)}{a(T)m(T)}$ is smaller in the area restricted Faustmann model, cf. comparison of (19) and (13). By assumption $m''(T) < 0$ and $a''(T) < 0$ and consequently the optimal rotation age becomes larger in the area restricted Faustmann model than in the area restricted CCF model.

3.4 The CCF Model Without an Area Restriction

As above we consider the problem of choosing the T that maximizes the value of a piece of land for forest use. However, now we relax the assumption of a restricted area being available for the forest, but as is inherent in the continuous cover forest, all ages are present in the forest from the beginning. For CCF without an area restriction the objective function needs to capture the return, in perpetuity, from harvesting from any specific area unit $a(T)$. The returns include not only the mature age class harvested at one point taking up all of $a(T)$, but also the returns from the area being used in part by other age classes harvested before T until it T years later is again occupied solely by a single age class of age T.

An example may illustrate the principle. Consider an age class T'. Once harvested the area of T', $a(T')$ is reallocated to younger trees, which when in turn they mature and are harvested, release their entire area, including the part of the $a(T')$ they took over from $a(T')$, which is then reallocated to other trees, and so on. Thus, a substantial part of any area, $a(T)$, at one point being occupied by only the age class T will over the course of the next T years be covered by several different generations. Consider the next to last generation, $T'-1$. It will be harvested a year later, and the area $a'(T-1)$ it overtook from the age class T' will then be re-distributed in part to the other trees that also took over part of $a(T')$. Thus, to capture all the value production from an area $a(T)$, we need to integrate both over time represented by t, but also over the T events of shifting land dynamic across the T age classes to be harvested, which each use part of $a(T)$.

The objective function for maximizing the value of a piece of land, $a(T)$, right after a tree of age T has been harvested, can thus be formulated as[10]:

$$
\begin{aligned}
MaxV_c = Max \Bigg[& \frac{m(T)a(T)e^{-rT}}{1-e^{-rT}} + \frac{\int_0^T \frac{a'(t)}{a(T)} m(T)a(T)e^{-r(T-t)}dt}{1-e^{-rT}} \\
& + \frac{\int_{t=0}^T \int_{i=t}^T \left(\frac{a'(i)}{a(T)-a(t)} m(T)a'(t)\, e^{-r(T-i)} \right) didt}{1-e^{-rT}} \\
& + \frac{\int_{t=0}^T \int_{i=t}^T \int_{j=i}^T \left(\frac{a'(j)}{a(T)-a(i)} m(T)a'(i)\, e^{-r(T-j)} \right) djdidt}{1-e^{-rT}} \\
& + \cdots + \frac{\int_{t=0}^T \int_{i=t}^T \int_{j=i}^T \cdots \int_g^T \int_{f=T}^T \left(\frac{a'(f)}{a(T)-a(g)} m(T)a'(g)\, e^{-r(T-f)} \right) dfdg \ldots djdidt}{1-e^{-rT}} \Bigg]
\end{aligned}
\quad (21)
$$

As in Sect. 3.1 the first term on the right hand side is the present value of an infinite series of identical rotations of an age class harvested at age T evaluated at $t = 0$. The remaining terms in (21) essentially capture the value of the intermediate returns to a specific area, $a(T)$, from other age classes using parts of the area until a new age class utilizes the entire area at age T. Because these intermediate returns also occur in all future rotations, we must multiply all these terms by the perpetuity factor, $1/(1 - \mathrm{e}^{-\mathrm{rt}})$. Notice that these intermediate returns are

[10] Remember that definite integrals are defined as limits on upper and lower boundaries on integration variables. Thus, in the third term we do not integrate over i = t but only a time approximately close to i and in (21) i = t in the third term is captured by the second term. Likewise, we only get approximately close to T at the upper limit of the integral. In this way we avoid double counting because we have a discontinuity in a(t) at T.

not thinnings, they result from harvesting of mature trees, which occupies a fraction of the area when harvested.

Looking at the intermediate return terms, the second term is the largest because it captures the value of the returns arising from re-allocation of the entire area $a(T)$ to all age classes younger than the one just harvested and older than the youngest one just established as replacement. Note that, in equilibrium, the aggregate change in area utilization of all age classes in any time period is exactly equal to $a(T)$, i.e. $\int_0^T a'(t)dt = a(T)$. From this it follows that the area is re-allocated according to the change of each age class, $a'(t)$, and that the intermediate return resulting from an age class arises at $T - t$. The implication of this is that the full area is allocated to age-classes that mature at different points in time.

The third and higher order terms in (21) all capture that once an age class, which has been occupying part of the area in focus, is harvested, the part of the $a(T)$ area is reallocated to the younger age classes on the area $a(T)$, including the youngest which will eventually take over again the entire area. For the first tree harvested after T the area re-allocated to age classes of younger age t is $a'(T)$, according to the same logic as for the second term in (21). However, the area available for re-allocation is only the part of $a(T)$ that is taken over by the age class, which is given by $a(T) - a(t)$. Note that because $\int_{i=t}^T a'(i)di = a(T) - a(t)$ we have, as for the second term, reallocated the full area part of $a(T)$ released at each intermediate time period between 0 and T. The fourth term captures the value of area available and reallocated at time i and so on. This chain of integrals continues until $f = T$ and this is captured by the last integral in last term. We note that as the time goes, fewer and fewer age classes will share a still smaller part of the area $a(T)$ considered. Thus, as the integral terms become more and more complex, the numerical value of these terms becomes ever smaller.

From (21) we can, in principle, derive a first-order condition with respect to T. However, such a condition would be excessively complex to derive and interpret because of the chain of integrals from the third term and onwards. We, therefore, conduct a second-order Taylor approximation of (21) around the point where $t = T$.[11] This approximation yields:

$$Max \left[V_c = \frac{e^{-rT} m(T)a(T)}{1 - e^{-rT}} + \frac{\int_0^T \frac{a'(t)}{a(T)} m(T)a(T)e^{-r(T-t)}dt}{1 - e^{-rT}} \right] \tag{22}$$

Here we keep in $a(T)$ to ease interpretation. Comparing the two expressions for V_c in (21) and (22) we see that the third term and onwards in (21) are excluded in (22). Thus, we include the two terms with largest value in (22) and exclude those with an area re-allocation significantly smaller than $a(T)$, and hence a smaller value production than for the second-order terms. The implication is that V_c in (21) is underestimating the true V_c for an unrestricted CCF. The size of this underestimation is illustrated for a numerical example in Sect. 4.

Comparing (22) with (1) the difference is the second, additional term in (22). Thus, the value of forest land, keeping T fixed, is largest in a CCF model because $\frac{\int_0^T \frac{a'(t)}{a(T)} m(T)a(T)e^{-r(T-t)}dt}{1-e^{-rt}} > 0$. Therefore, a CCF has a higher present value per area than the Faustmann forest, ceteris paribus. Notice that as T increases, the area occupied by the forest

[11] A requirement for the Taylor series approximation is continuity of the objective function with respect to a, m and t. This is also a requirement in most other economic models used for optimization (Varian 1992), including the ones earlier in this paper, and it will hold for most functional forms that can be imagined in the current setting.

increases in the unrestricted CCF model, but not in the unrestricted Faustmann forest. Notice also, that the value of CCF is larger than for the Faustmann model, but this is also contingent upon a forest already being established, giving an early return.

We may now derive a first-order condition of (22) and this condition is given by:

$$\frac{m'(T)a(T)+a'(T)m(T)}{m(T)a(T)} = r\left(1+\frac{e^{-rT}}{1-e^{-rT}}+\int_0^T \frac{a'(t)}{a(T)}e^{rt}dt \right.$$
$$\left. +\frac{1}{1-e^{-rT}}\int_0^T \frac{a'(t)}{a(T)}e^{-r(T-t)}dt\right)-\frac{a'(T)}{a(T)}e^{rT}-\frac{m'(T)}{m(T)}\int_0^T \frac{a'(t)}{a(T)}e^{rt}dt \qquad (23)$$

Now we interpret (23) by comparing the solution for CCF in (23) with the solution in the traditional Faustmann forest in (4). We start by noting that the Faustmann capital cost of forest production for infinite time. $r\left[1+\frac{e^{-rT}}{1-e^{-rT}}\right]$, is included in both models. However, for CCF the capital cost term has two additional components given by $r\int_0^T \frac{a'(t)}{a(T)}e^{-r(T-t)}dt/(1-e^{\cdot rT})$ and $r\int_0^T \frac{a'(t)}{a(T)}e^{rt}dt$. Because these are positive, they make the capital cost measure larger, and thereby push the rotation age down.

However, the components reflecting the differences in the marginal value of production of the CCF forest also appear in (23). They essentially belong on the LHS along with the other marginal return components but we move them to the RHS for ease of comparison across the four results. These terms relate to the relative change in area utilization, $e^{rT}\frac{a'(T)}{a(T)}$, and the marginal change in the present value of the production across the age classes, $\frac{m'(T)}{m(T)}\int_0^T \frac{a'(t)}{a(T)}e^{rt}dt$. The two components are negative, when placed here on the RHS, and this tends to make the rotation age larger because $a''(T) < 0$ and $m''(T) < 0$. Thus, comparing rotation age between CCF and Faustmann forest without area restrictions is ambiguous and the final effect will depend on the relative size of the terms added and subtracted (i.e. on the functional forms of $a(T)$ and $m(T)$ and on the size of the discount rate). We analyze this issue by using simulations for reasonable specific functional forms in Sect. 4.3.

Note that the first-order condition in (23) is based on a second-order Taylor approximation of the true present value of a CCF. Including terms of third and higher order would increase the capital cost and this would tend to make the rotation age smaller. However, at the same time the marginal value of forest production becomes larger and this tends to increase the rotation age. Thus, compared to the original Faustmann model using the full objective function in (21) has the same ambiguous effects as when using a second-order Taylor approximation. To conclude we cannot say anything definite about the rotation age when comparing the Faustmann model and continuous cover model without an area restriction.

4 Numerical Illustrations

In this section we will illustrate the analytical solution from Sect. 3 by the use of reasonable functional forms. We will focus on numerical examples of assessing the optimal rotation ages for the different models, and we will show likely outcomes. We have also undertaken

Table 1 Parameters in the Gompertz function

	$m(t)$	$a(t)$
G	0.15	0.10
K	250,000	0.02

an assessment of the approximation error implied by the Taylor series approximation in the fourth model variant in Sect. 3.4.

4.1 A Numerical Example

Most advanced empirical forest growth models account for multiple generations growing together under competition. However, different species are too complex to be of use for illustrating the models in this paper. Therefore, we assume functional forms for $a(T)$ and $m(T)$ that are simple but have parametric values of reasonable sizes in order to evaluate the impact of different model assumptions. For both functions we choose a variant of a Gompertz function with the general form for growth $x'(t)$, where $x(t)$ can be either $a(T)$ or $m(T)$[12]. Thus, we have that is:

$$x'(t) = g_x \ln (K_x / x(t)) \,. \tag{24}$$

where K_x is a parameter giving the maximum level of x (carrying capacity) and g_x is the initial growth rate. We chose values of the parameters g_x and K_x for the two functions that resembled realistic sizes and patterns of the growth in beech under favorable site and productivity conditions in Denmark (Nord-Larsen et al. 2009). The various values are shown in Table 1.

We analyzed the results for a number of alternative parameter values to test the sensitivity of the results to these. In addition we selected an interest rate r of 4%, but we also undertook a range of sensitivity analyses for this parameter (see Sect. 4.2 below). The interest rate selection is based on the observation by past studies that equilibrium return rates and hence discount rates for long rotation forestry system are typically quite low (below 4%) in the Nordic countries (Brukas et al. 2001; Lundgren 2005; Thorsen 2010) and North America (Washburn and Binkley 1990, 1993). Short rotation plantations may have higher equilibrium returns (Brukas et al. 2001).

4.2 Optimal Rotation Age

In Figs. 2 and 3, we show corresponding solutions to the optimal rotation age problems in the four different models (from Sects. 3.1 to 3.4).

We see how the rotation age is lower in the area restricted CCF model than in the corresponding Faustmann model, as predicted in Sect. 3.3. For the unrestricted forest models, we were not able to find an unambiguous result in the Sect. 3.4, but the numerical results here shows that the rotation age in the CCF model is higher than the rotation age in the Faustmann model. Note how the RHS of the area restricted CCF model has a significantly different form; and remember that this is caused by the different ordering of the marginal value and marginal cost terms for this solution.

[12] This function is chosen as the driving factor for both m and a is the biological growth—and limits imposed by competition for resources. For realistic assumptions of price development (e.g. that price increase by dimension, but only until dimensions are too large to handle) this general functional form is not contradicted. Thus, by working with a Gompertz function it is generalized to a relatively flexible form.

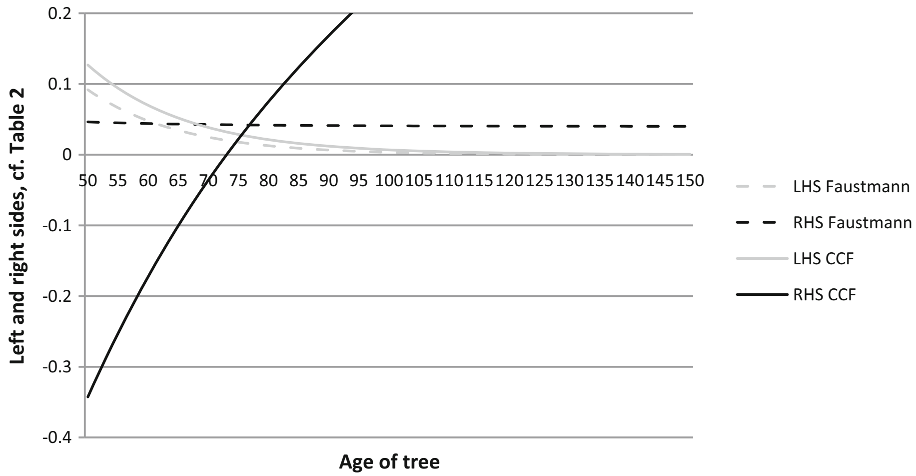

Fig. 2 First order conditions for the *area unrestricted* model. We find a rotation age that is longer for the CCF solution than for the Faustmann solution. *CCF* refers to continuous cover forestry

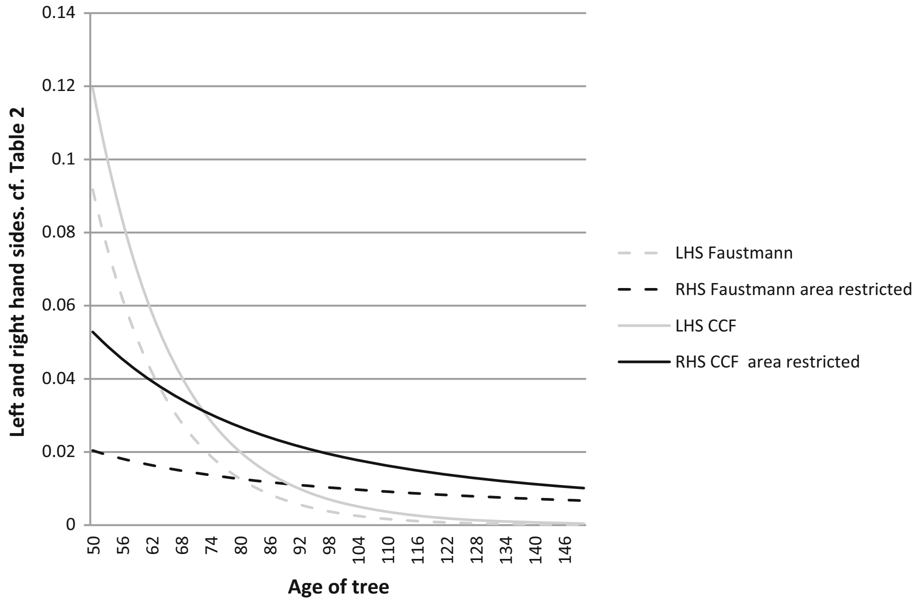

Fig. 3 First order conditions for the *area restricted* model. We find a rotation age that is shorter for the CCF solution than for the Faustmann solution. *CCF* refers to continuous cover forestry

Our numerical result holds for different interest rates and parameter assumptions. We found that it is most sensitive to g for $a(t)$—and hence the larger g, the closer are the two rotation ages. This reflects the fact that the faster the initial growth in area utilization, $a'(t)$, of a new tree, the less the time needed to take over the entire area $a(T)$ and the smaller becomes the differences between the two forestry systems.

Table 2 The size of the three first RHS-terms of Eq. (21) and the pairwise ratios between them for various discount rates

Interest rate (%)	1st term	2nd term	3rd term	Ratio 2:1	Ratio 3:1	Ratio 3:2
0.50	7058	8816	578	1.25	0.08	0.07
1	2833	4451	318	1.57	0.11	0.07
2	878	2240	188	2.55	0.21	0.08
3	346	1482	145	4.28	0.42	0.10
4	148	1091	123	7.40	0.83	0.11
5	65	852	109	13.16	1.68	0.13
6	29	691	99	23.99	3.45	0.14
7	13	576	92	44.71	7.18	0.16

4.3 The Second-Order Taylor Approximation

A main assumption in Sect. 3.4 is the second-order Taylor approximation applied in (22). For a given rotation age ($T = 80$), we estimate the size of the three first RHS-terms of (21) for different interest rates and the results are shown in Table 2 above.

The first thing we notice is that the second term is much larger than the first term. Thus the value of a CCF will be much larger than a traditional Faustmann forest. The size ratio depends on the explicit functional form of $a'(t)$—the faster the initial increase in area utilization, the smaller the ratio. Remember, however, that V_F and V_C are not directly comparable as the requirement for the CCF model is that there are already trees on the area which can give the return captured by the second term. Thus V_C is the present value of an existing forest with multiple age classes rather than a soil expectation value of bare land.

Next we notice that the third term is much smaller than the second term—7–14% for the chosen interest rates. We also see that the higher the interest rate, the larger is the approximation error, the reason being that the intermediate return in the integrals (for t between $=$ and T) weights more relative to the final harvest at T multiplied by the eternity factor for high interest rates. The error is sufficient low for the approximation to be reasonable, in particular as relevant interest rates, as explained above, are typically below 4%. Other functional forms were tried, and the results described above hold for a variety of assumptions.

5 Discussion

Forest managers and researchers in silviculture all over Europe have looked to German traditions for CCF and management of mixed species stands. These management systems are often considered more sustainable than the clear-cut based plantation systems usually associated with Faustmann forest economics. In Germany, many forests are managed more or less according to continuous cover principles and rotation ages are often found to be higher than the ages usually implied by the Faustmann criterion. There may be many reasons for this and one reason is the long-standing debate about whether or not to use interest rates in decision making (Möhring 2001), but of course other things like different growth patterns, harvesting rules, returns and costs matter. However, an obvious question is whether it is an optimal feature implied by the continuous cover management system per se. Likewise, many observers have pointed out that the volumes and values tied up in German forests (managed according to near-natural principles) are much higher than typical average volumes and values in forests managed according to Faustmann's rules. Again the explanations may be many, but it is relevant to investigate if it is a result to be expected because of the different management dynamics per se (Möhring 2001). In other words, is the higher capital stock observed in CCF in fact optimal, seen from a traditional economic objective, or are other explanations needed?

The models presented in this paper address one particular aspect of the CCF systems, which we find intriguing, and this aspect is the way different age classes over time share the land area. We stress that the models are abstract, and based on rather crude assumptions—much like the Faustmann model. Nevertheless, we believe the models offer some insights into economic aspects of CCF. This may help pinpoint some of the likely differences between optimally managed CCF and the classic even-aged forest. Including area dynamics essentially results in two consequences—a more efficient area use through a different area re-allocation and a weighting of the productivity by the area use of each age class.

We showed formally that the value of the CCF will always be larger than or equal to the value of the Faustmann forest for any given rotation age. This conclusion holds both with and without an area restriction. The explanation is that larger trees with a higher volume per area unit cover a higher proportion of the area in the CCF, thereby utilizing the area more efficiently. Therefore, CCF must have a larger standing volume per hectare in the optimally managed forest.

The optimal rotation age in a CCF forest is likely to be affected too. For the area unrestricted models we find that the optimal rotation age may be shorter or longer compared with the Faustmann rotation ages. In our numerical illustration (Table 1; Fig. 3), we find that the rotation age is higher for realistic functions of $a(T)$ and $m(T)$ in the continuous cover model. For the area restricted models, the results are unambiguous, and we find that the optimal rotation age of the CCF will always be shorter than the corresponding area-restricted Faustmann rotation age. This is a result of the shadow cost of land being higher under the more productive continuous cover system. The difference between the systems in the area unrestricted case results from the CCF growing in area use as the rotation age grows.

Except for including different area utilization, the two models for CCF have the same assumptions as the basic Faustmann models. The results indicate that the higher standing volume as seen in practical management of CCF may be optimal under these assumptions.

The simple numerical examples also show that longer rotations could be economically optimal in the CCF case. This result does not mean that very high rotation ages are optimal, but in a CCF it certainly sheds new light on more than 100 years of debate among practitioners of various forest management schools. It has sometimes been argued that due to the forest area being restricted, observed high rotation ages are justified by the longer maximum sustained yield rotations obtained under Faustmann with an area restriction (Möhring 2001). However, the area restricted CCF model we present has an unambiguously lower optimal rotation age compared to the area restricted Faustmann model. In the real world other goals than the pure economic objectives could be used, e.g. sustainability constraints, tradition or other reasons (Möhring 2001). As Hartman (1976) shows this may result in ambiguous consequences on the rotation age—depending on the specification of the amenity.

5.1 Limitations, Caveats and Future Improvements

One obvious caveat in this work is the Taylor series approximation used in our unrestricted continuous cover forest model, which leads to an underestimation of the value of the CCF. For our numerical example, results showed that the underestimation was in the range 7–14% depending on the interest rate, and at the lower end for the most likely and reasonable interest rates. However, for the first order conditions, this affects the right-hand-side in ambiguous ways—as also for the already included additional terms. Consequently the effect on rotation age is likely to be small.

In continuous cover systems, a large number of individuals in the youngest age classes are typically being reduced through selective thinning or natural mortality as time passes. These individuals often grow underneath older trees and take over much of the released area, but many of the trees also die early due to competition. We have not explicitly modelled this aspect in the paper, but have included it in the interpretation of $a(t)$ as being the area of the age class rather than of a single tree. We use a rather crude approximation of the $a(t)$ function. Ideally, the $a(t)$ should be based on data better approximating the ecological aspects of the forest growth, taking into account that the area function, $a(t)$, may represent more than the area as such, and for example include the productive factors related to the area like light, nutrients and precipitation. Likewise, extensions could include or reflect a time-lag for area utilization, reflecting that when a tree is harvested, it may take a little while before the productive factors are utilized completely by neighbouring trees. More complicated is the introduction on between-tree competition for area and resources, and its effect on $m(t)$. Such extensions would call for numerical methods for analysis as e.g. Meilby and Nord-Larsen (2012), as mathematical tractability will be lost.

Another caveat is that ignoring thinnings may perhaps bias towards a somewhat larger difference between the CCF and the Faustmann models. In the even aged Faustmann forest, young forest stands include many more tree seedling and young trees than will be around for maturity. The surplus trees are in some management systems harvested in thinnings along the life of the stand, resulting in cash flows (sometimes negative cash flows, i.e. costs). These surplus trees utilise area and resources not utilised by the final harvest trees before later in their life, and thus represent an attempt to increase the efficiency of area use. Nevertheless, in the CCF as modelled here, the efficiency will be greater as the area following a final harvest is not allocated only to a large host of new tree seedlings of which many will never produce much value, but rather the bulk of area is allocated to larger trees producing much more valuable wood over the time period.

Related to this caveat, one would be right in questioning if $m(T)$ and $a(T)$ are likely to be the same in two so different management systems with most likely quite different competition dynamics. This is a simplifying assumption made in this paper, which is necessary to be able to say something precise about the role of the differences in land use dynamics. It will remain an empirical question, probably one specific to each the different kinds of CCF and Faustmann management practiced, if such differences exist and if they will matter for decisions. As Faustmann management usually focus much on regulating competition across species to the benefit of the final harvest individual, one could expect that $m(T)$ for a Faustmann tree would be higher for any t than for the CCF case, and likewise for $a(T)$; this would leave the impact on our results ambiguous.

Turning to other assumption, we can use existing knowledge on the effects of some of these aspects in the Faustmann model (see Newman 2002; Amacher et al. 2009 for reviews), to draw a few conclusions about the possible results and effects of including them. First, we may look at regeneration costs as they are usually expected to be lower for continuous cover forest. We know that, in the Faustmann case, lower regeneration costs imply a shorter rotation age (cf. e.g. Amacher et al. 2009). Thus, we would expect including costs to further increase the value of the CCF and reduce the optimal rotation age relative to the Faustmann forest. Second, higher harvesting cost are expected in CCF than in even-aged forest (see e.g. Price and Price 2006; Amacher et al. 2009), and we know from the Faustmann solution that this prolongs the rotation. Thus, the effect of including higher harvesting cost in CCF is likely to prolong rotations of the CCF relative to the Faustmann forest.

A final comment is justified regarding the issue of transitions from one forest system to another. Our paper has analysed equilibrium versions of the CCF and Faustmann management systems, and has not considered the difficult, but relevant question if and how, it would be optimal to move from one system to another. This transition has been discussed in the practical debate, but we leave it for further research to discuss and analyse this. To be meaningful, such analyses are likely best undertaken as numerical and empirically well informed analyses of actual management systems and situations.

6 Concluding Remarks

The movement towards CCF or near-natural forest management is ongoing in a number of European countries, and is globally linked to sustainable forest management. CCF has a potential to change rather drastically the way the forest production apparatus works. This will also have strong implications for the way economists should analyze these forest systems in order to suggest reliable management principles with transparent economic outcomes.

In the literature, very few attempts have been conducted to elucidate analytically one major and intriguing difference between the even-aged forest management system and the CCF: the area utilization dynamics. The former brings about cyclical cash flows originating from the harvesting of trees, which can safely be considered as independent decision units. Under CCF the area released when harvesting a tree is allocated to a number of trees of varying ages and productive capacities already in place or soon to be in place.

The present paper only attempts to capture parts of these different area dynamics in models simple enough to answer some of the key questions in forest management. How should a forest be valued? And when should a tree be harvested? We develop models for CCF which captures the key dynamic by explicit accounting for area reallocation and for weighting the productivity of each age class by the area occupied. Using these models we are able to show that optimally managed continuous cover forests with and without an area restriction will always have a larger standing value than the corresponding Faustmann based normal forest. This is in accordance with empirical observations discussed in the literature. The unrestricted models allows for both shorter and longer rotation ages in the CCF case compared with the Faustmann case, whereas in the area restricted case the rotation age is unambiguously shorter under the CCF regime. We provide an illustration using simple functional forms resembling very roughly the growth potential in a beech forest in Denmark.

The models are rather simple and crude, because we focus explicitly on the different area utilization dynamics to allow for comparisons with the well-established results of the Faustmann equation. Many extensions can be conducted to make this framework closer to the empirical reality of silviculture, and we have discussed some of the likely impacts of such extensions.

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